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Propagators and WKB-exactness in the plane wave limit of $AdS \times S$

Danilo E. Diaz and Harald Dorn
Humboldt-Universität zu Berlin, Institut für Physik
Newtonstr. 15, D-12489 Berlin, Germany
E-mail: ddiaz@physik.hu-berlin.de, dorn@physik.hu-berlin.de

Abstract: Green functions for the scalar, spinor and vector fields in a plane wave geometry arising as a Penrose limit of $AdS \times S$ are obtained. The Schwinger-DeWitt technique directly gives the results in the plane wave background, which turns out to be WKB-exact. Therefore the structural similarity with flat space results is unveiled. In addition, based on the local character of the Penrose limit, it is claimed that for getting the correct propagators in the limit one can rely on the first terms of the direct geodesic contribution in the Schwinger-DeWitt expansion of the original propagators . This is explicitly shown for the Einstein Static Universe, which has the same Penrose limit as $AdS \times S$ with equal radii, and for a number of other illustrative cases.

Keywords: Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence.

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1. Introduction

Recently, the study of strings in plane wave backgrounds has received a lot of attention. These activities are due to the observation [1] that, via a special limit of the standard AdS/CFT correspondence, string theory on a certain plane wave background corresponds to a large R-charge subsector of the $\mathcal{N} = 4$ super Yang-Mills gauge field theory. In contrast to the original $AdS_5 \times S^5$, in these plane wave backgrounds the exact quantization of strings is known. This allows tests of the correspondence including genuine stringy properties.

Related to these considerations also the field theoretical properties of plane wave backgrounds became relevant. In particular the propagators, both bulk to bulk and bulk to boundary, for $AdS_5 \times S^5$ and for the plane wave arising in a Penrose limit should play a crucial role in understanding the degeneration of the holographic picture from a 4- to a 1-dimensional boundary. In spite of several attempts [2, 3, 4], this issue is up to now not completely clarified.

The scalar propagator in the relevant plane wave has been constructed for generic mass values via direct mode summation in [5]. In addition, there have been observed structural similarities with the flat space propagator and their possible role in guessing the higher spin propagators was stressed. The alternative route via the limiting behavior of the $AdS_5 \times S^5$ propagator was taken in [6] for the conformally coupled scalar.

In the present paper we want to address the propagators in the plane wave background along the line of the Schwinger-DeWitt construction. This technique, based on an expansion near the light cone, has a long history. It has been successfully applied to the propagator construction in various specific backgrounds as well as to issues related to near light cone and anomaly problems in generic backgrounds. It lies at the heart of most regularization techniques of QFT in curved spaces (see, e.g. [7]). Our aim here is to explain the above mentioned structural similarities to the flat space case by the termination of the underlying WKB expansion and to make progress in the explicit construction for higher spin cases. We will also explore the alternative approach to derive the plane wave propagators as a limiting case of propagators in spaces which in a Penrose limit yield the plane wave under discussion. For this we relate our results to information on propagators in Einstein Static Universe (ESU) available in the literature.

The paper is organized as follows. After collecting some preliminaries on plane waves, AdS and ESU in section 2, we apply in section 3 the Schwinger-DeWitt technique to construct the plane wave propagators for the scalar, spin 1/2 and the spin 1 gauge field propagator. Section 4 is devoted to some comments on known results on propagators and Schwinger-DeWitt kernels in ESU and their relation via a Penrose limit to the spin 0 and spin 1/2 results of the previous section. Section 5 reproduces for the conformal flat cases of $AdS_{p+1} \times S^{q+1}$ and Weyl invariant coupling of scalars the results of [6] within the Schwinger-DeWitt technique and comments on the role of direct and indirect geodesic contributions before and after taking the Penrose limit to the plane wave. In addition we find a relation between the propagators in a special conformal non-flat situation and a flat one via a contour integral. This special non-flat situation concerns just the special values for curvature radii and mass values which still allowed an explicit summation in [6]. We end with a summary and some conclusions. Various technical details are collected in a set of appendices.

2. Penrose limit: plane wave background

The particular plane wave background to be considered is the conformally flat one obtained as a Penrose limit of $AdS_5 \times S^5$ [8, 1] with equal radii, although at some stages the results can be adapted to other dimensions by just varying the number of transverse directions $\vec{x}$. The line element is given by

$$ds^2 = 2dudv - \vec{x}^2 du^2 + d\vec{x}^2 \,. \tag{2.1}$$

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As noticed by Penrose [9], this limit is nothing but an adaptation to pseudo-riemannian manifolds of the standard procedure of taking tangent space limit, the main difference being that when applied to a null geodesic it results in curved space, namely a plane wave. One could as well end up with flat space, but the generic situation is a plane wave. It is this zooming into the neighborhood of the null geodesic what gives the Penrose limit a local character.

Recently, Penrose limits of a whole variety of space-times have been thoroughly studied (see, e.g. [2] and reference[13] therein). The particular plane wave metric (2.1) together with a RR-flux corresponds to a maximally supersymmetric solution of type-II-B SUGRA, as first found by BFHP [8]. This very same type-II-B SUGRA background can also be obtained as a Penrose limit of the less supersymmetric $AdS_5 \times T^{1,1}$ [10], and surely from many other backgrounds. Now, as far as one is interested only in the metric, the spacetime with the same Penrose limit, which ought to be considered the conceptually simplest one, is the Einstein Static Universe ESU_{10}. In parts of the following discussion we will benefit from this fact.

2.1 Anti-de Sitter × sphere

Let us start with AdS_{p+1} in global coordinates and with the $(q+1)$-sphere parametrized in terms of a $(q-1)$-sphere (a is the common radius of AdS and the sphere)

$$ds^2_{AdS_{p+1}\times S^{q+1}} = a^2\left(-dt^2\cosh^2\rho + d\rho^2 + \sinh^2\rho d\Omega^2_{p-1} + \cos^2\theta d\psi^2 + d\theta^2 + \sin^2\theta d\Omega'^2_{q-1}\right). \tag{2.2}$$

Now one focuses on the immediate neighborhood of a null geodesic that remains at the center of AdS_{p+1} while it wraps an equator of S^{q+1}, say $t = \psi = u$ (affine parameter along the null ray) and $\rho = \theta = 0$. Introducing local coordinates

$$t = u \qquad \psi = u + \frac{v}{a^2} \qquad \rho = \frac{x}{a} \qquad \theta = \frac{y}{a} \tag{2.3}$$

and expanding in inverse powers of the radius, one gets

$$ds^2_{AdS_{p+1}\times S^{q+1}} = 2dudv - (x^2+y^2)du^2 + dx^2 + x^2 d\Omega^2_{p-1} + dy^2 + y^2 d\Omega'^2_{q-1} + O(a^{-2}), \tag{2.4}$$

so that in the limit $a \to \infty$, blowing up the neighborhood and collecting the flat transverse directions into $\vec{x}$, one ends up with the plane wave metric (2.1).

2.2 Einstein static universe

To see that the same plane wave results from ESU_n, topologically $R \times S^{n-1}$, let us conveniently parametrize the $(n-1)$-sphere in terms of a $(n-3)$-sphere

$$ds^2_{ESU_n} = a^2(-dt^2 + d\Omega^2_{n-1}) \tag{2.5a}$$

$$= a^2\left(-dt^2 + d\alpha^2 + \cos^2\alpha\, d\beta^2 + \sin^2\alpha\, d\Omega^2_{n-3}\right). \tag{2.5b}$$

This time the null geodesic will be the one given by $t = \beta = u$ (affine parameter along the null ray) and $\alpha = 0$, and the local coordinates in its neighborhood

$$t = u \qquad \beta = u + \frac{v}{a^2} \qquad \alpha = \frac{r}{a}. \tag{2.6}$$

Then, expanding the line element

$$ds^2_{ESU_n} = 2dudv - r^2du^2 + dr^2 + r^2 d\Omega^2_{n-3} + O(a^{-2}) \tag{2.7}$$

and letting $a \to \infty$ one gets again the plane wave metric (2.1).

That both Penrose limits give the same metric can be easily understood if one remembers that there is a conformal map that allows for a Penrose diagram for $AdS_{p+1} \times S^{q+1}$. Defining $\tan\vartheta \equiv \sinh\rho$ in (2.2), one obtains that both metrics are related by[1]

$$ds^2_{AdS_{p+1}\times S^{q+1}} = \frac{1}{\cos^2\vartheta} ds^2_{ESU_{p+q+2}} . \tag{2.8}$$

Now, in the local coordinates (2.3) near the null geodesic at the center of Ad_{p+1} we have $\vartheta = \frac{x}{a} + O(a^{-3})$ and the conformal factor $\cos^{\pm 2}\vartheta = 1 + O(a^{-2})$, therefore up to $O(a^{-2})$ both metric are equivalent, i.e. the r.h.s. of (2.4) holds for both backgrounds. Consequently, in the limit $a \to \infty$ the resulting metrics coincide. Notice that this time we had a different parametrization of the $(n-1)-$sphere in ESU_n and different local coordinates, but their departures are scaled away in the plane wave limit resulting in the same spacetime. This is again a manifestation of the inherent locality of the Penrose limit.

3. Propagators in the plane wave

The scalar Feynman propagator in the plane wave background (2.1) has already been obtained by explicit summation of the eigenmodes in recent works [5, 2]. Here we will treat it differently using the Schwinger-DeWitt technique which admits a readily generalization to the spinor and vector fields.

3.1 Scalar propagator: Schwinger-DeWitt proper-time

The scalar Feynman propagator in a curved background is the solution of the wave equation with a point-like source

$$\left(\Box - m^2\right) G(x, x') = \delta(x, x') \tag{3.1}$$

together with appropriate boundary conditions. Here $\delta(x, x')$ denotes the invariant δ-function. The Schwinger-DeWitt proper-time representation for the Feynman propagator [11], which incorporates the Feynman boundary conditions by the $i0^+$ prescription, is based on the formal solution

$$\frac{1}{\Box - m^2 + i0^+} = -i \int_0^\infty ds\, e^{-is\,m^2 - s0^+}\, e^{is\,\Box} . \tag{3.2}$$

The Schwinger-DeWitt kernel (the kernel of the exponentiated operator),

$$K(x\,, x' \mid s) \equiv \langle x \mid e^{is\,\Box} \mid x' \rangle = e^{is\,\Box}\delta(x, x') \tag{3.3}$$

[1] Obviously, there is an obstruction to this argument if the null geodesic, on which one focuses in the Penrose limit, reaches the boundary of $AdS \times S$ where the conformal factor becomes singular. It is precisely in this situation when the null geodesic is totally contained in AdS and the Penrose limit of $AdS \times S$ gives just Minkowski space.

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satisfies the following "Schrödinger equation" and initial condition

$$(i\partial_s + \Box)\, K(x, x' \mid s) \,=\, 0 \tag{3.4a}$$

$$K(x, x' \mid 0) \,=\, \delta(x, x')\,. \tag{3.4b}$$

A WKB-inspired ansatz for the solution, meant to be only an asymptotic one, is

$$K(x, x' \mid s) = \frac{i}{(4\pi i s)^{\frac{d}{2}}}\, \triangle^{\frac{1}{2}} e^{\,i\sigma/2s}\, \Omega(x, x' \mid s) + \cdots \tag{3.5}$$

where $\sigma(x, x')$ is the geodetic interval (one half the geodesic distance squared between the two points),

$$\triangle(x, x')[g(x)g(x')]^{\frac{1}{2}} \equiv -\det\left(-\frac{\partial^2 \sigma}{\partial x^\mu \partial x'^\nu}\right) \tag{3.6}$$

is the Van Vleck-Morette determinant (an important improvement of the WKB ansatz). $\Omega(x, x' \mid s)$ has a power expansion in the proper time s

$$\Omega(x, x' \mid s) = \sum_{n=0}^{\infty} (is)^n\, a_n(x, x')\,, \tag{3.7}$$

whose coefficients $a_n(x, x')$ are regular functions in the coincidence limit $x \to x'$, and finally the ellipsis stands for indirect geodesic contributions. The coefficients, sometimes referred to as HaMiDeW coefficients, must satisfy the recursion relation

$$(n+1)\, a_{n+1} + \partial^\mu \sigma\ \partial_\mu a_{n+1} = \triangle^{-\frac{1}{2}}\, \Box\, (\triangle^{\frac{1}{2}} a_n) \tag{3.8}$$

starting with $\partial^\mu \sigma\ \partial_\mu a_0 = 0$ and $a_0(x, x) = 1$. For the present scalar case, the chain of HaMiDeW coefficients trivially starts with $a_0(x, x') = 1$.

Now we are in position to apply this construction to the plane wave background. From the geodetic interval between two generic points (see appendix C) one obtains the Van Vleck-Morette determinant. The important ingredients are

$$g(x) \,=\, -1 \tag{3.9a}$$

$$\sigma(x, x') \,=\, (u - u')\left[v - v' + \frac{\vec{x}^2 + \vec{x}'^2}{2}\, \cot(u - u') - \vec{x}\cdot\vec{x}'\, \csc(u - u')\right] \tag{3.9b}$$

$$\triangle(x, x') \,=\, \left[\frac{u - u'}{\sin(u - u')}\right]^{d-2}. \tag{3.9c}$$

With this at hand, one can check that $\triangle^{\frac{1}{2}}(x, \cdot)$ is harmonic, i.e. $\Box\, \triangle^{\frac{1}{2}}(x, \cdot) = 0$, because $\triangle(x, \cdot)$ is a function only of u and the inverse metric has $g^{uu} = 0$, so that the recurrence relations are satisfied by $a_n(x, x') = \delta_{0,n}$. Thus the only non-zero coefficient in the expansion is just the first one. That is why we say that the scalar Schwinger-DeWitt kernel in the plane wave background is leading-WKB exact. The kernel and the Green function, after performing the proper time integral, are then given by[2]

$$K(x, x' \mid s) \,=\, \frac{i \triangle^{\frac{1}{2}}}{(4\pi i s)^{\frac{d}{2}}}\, e^{\,i\sigma/2s} \tag{3.10a}$$

[2] As usually, the Feynman Green function should be understood as the boundary value of a function which is analytic in the upper-half σ plane, so that in fact $\sigma + i0^+$ is meant in what follows.

$$G(x,x') = \frac{-i\pi\triangle^{\frac{1}{2}}}{(4\pi i)^{\frac{d}{2}}} \left(\frac{2m^2}{\sigma}\right)^{\frac{d-2}{4}} H^{(2)}_{\frac{d}{2}-1}\left(\left[-2m^2\sigma\right]^{\frac{1}{2}}\right). \tag{3.10b}$$

One can get Minkowski space by rescaling $u \to \mu u, v \to v/\mu$ and letting μ go to zero. The effect of this in (3.9), (3.10) is $\triangle \to 1$ and $2\sigma \to 2(u-u')(v-v') + (\vec{x}-\vec{x}')^2$ and

$$K_M(x,x' \mid s) = \frac{i}{(4\pi i s)^{\frac{d}{2}}} e^{i\sigma/2s} \tag{3.11a}$$

$$G_M(x,x') = \frac{-i\pi}{(4\pi i)^{\frac{d}{2}}} \left(\frac{2m^2}{\sigma}\right)^{\frac{d-2}{4}} H^{(2)}_{\frac{d}{2}-1}\left(\left[-2m^2\sigma\right]^{\frac{1}{2}}\right). \tag{3.11b}$$

The difference between the two results, apart from the fact that the geodetic interval is of course different, is that for the plane wave we get a nontrivial Van Vleck-Morette determinant. The analogy with the Minkowski case observed in [5] is thus fully explained by the leading-WKB exactness of the plane wave background. The coincidence limit of our results, where the coefficients become local functions of curvature invariants [11, 12], is consistent with the fact that for the plane wave background there are no non-vanishing curvature invariants [13].

Finally, for the massless scalar one can take the massless limit in both expressions to get[3]

$$D(x,x') = \frac{-i\,\Gamma(d/2-1)}{2(2\pi)^{d/2}} \triangle^{\frac{1}{2}} \left(\frac{1}{\sigma}\right)^{\frac{d-2}{2}} = \frac{-i\,\Gamma(d/2-1)}{2(2\pi)^{d/2}} \left(\frac{1}{\Phi}\right)^{\frac{d-2}{2}} \tag{3.12}$$

$$D_M(x,x') = \frac{-i\,\Gamma(d/2-1)}{2(2\pi)^{d/2}} \left(\frac{1}{\sigma}\right)^{\frac{d-2}{2}}. \tag{3.13}$$

3.2 Spinor field: leading-WKB-exactness

One might guess that the similarity with Minkowski space kernel and Green function still holds for higher spin fields. Now we will turn our attention to the spin $\frac{1}{2}$ case.

The spinor Green function is now a bi-spinor which satisfies the Dirac equation with a point-like source

$$[\gamma^\mu(x)\nabla_\mu + m]\,\mathbb{S}(x,x') = \delta(x,x')\mathbb{I}, \tag{3.14}$$

where $\gamma^\mu(x)$ are the curved space Dirac matrices and ∇_μ is the spinor covariant derivative (see appendix B).

To apply the Schwinger-DeWitt technique one introduces an auxiliary bi-spinor $\mathbb{G}(x,x')$ defined by

$$\mathbb{S}(x,x') = (\gamma^\mu(x)\nabla_\mu - m)\,\mathbb{G}(x,x') \tag{3.15}$$

to obtain the following wave equation for $\mathbb{G}(x,x')$

$$\left(\Box - \frac{R}{4} - m^2\right)\mathbb{G}(x,x') = \delta(x,x')\mathbb{I}, \tag{3.16}$$

where R is the scalar curvature.

[3] The geometrical meaning of the quantity Φ is explained in appendix A.

Now one can apply the Schwinger-DeWitt construction as in the scalar case, but this time the auxiliary Green function $\mathbb{G}(x,x')$, the kernel $\mathbb{K}(x,x' \mid s)$ as well as the HaMiDeW coefficients $\mathbb{A}_n(x,x')$ are bi-spinors and the recurrence relations (3.8) involve the spinor covariant derivative. One starts with $\mathbb{A}_0(x,x') = \mathbb{U}(x,x')$, the spinor parallel transporter along the geodesic connecting the two points (appendix B).

For the plane wave background (2.1) one can check that again the recurrence relations are satisfied by $\mathbb{A}_n(x,x') = \delta_{n,0}\mathbb{U}(x,x')$, the reason being that $\triangle^{\frac{1}{2}}(x,\cdot)\mathbb{U}(x,\cdot)$ is harmonic, with respect to the spinor D'Alembertian.

Therefore, the spinor kernel and the spinor auxiliary Green function are leading-WKB exact and can be written in terms of the respective scalar quantities

$$\mathbb{K}(x,x' \mid s) = K(x,x' \mid s)\,\mathbb{U}(x,x')\,, \tag{3.17a}$$

$$\mathbb{G}(x,x') = G(x,x')\,\mathbb{U}(x,x')\,. \tag{3.17b}$$

Flat space results can also be recovered as in the scalar case, taking into account that in the limit $\mathbb{U}(x,x') \to \mathbb{I}$. The similarity with flat space result is still present, the only additional nontrivial piece being the spinor geodesic parallel transporter, and is better appreciated in terms of the kernel and the auxiliary Green function, so we do not show the explicit expression for $\mathbb{S}$.

3.3 Vector field: next-to-leading-WKB-exactness

Let us examine the Maxwell field. Now we have additional complications due to the gauge freedom, so we add a gauge fixing term $-\frac{1}{2\xi}(\nabla_\mu A^\mu)^2$ in the action to get an invertible differential operator

$$\left[g_{\mu\rho}\Box - R_{\mu\rho} - (1-\xi^{-1})\nabla_\mu\nabla_\rho\right] G^{\rho}_{\ \nu'}(x,x') = \delta(x,x')g_{\mu\nu'}(x)\,, \tag{3.18}$$

where the Ricci tensor $R_{\mu\nu}$ arises from the commutator of the covariant derivatives. Its only non-vanishing component in the plane wave geometry is $R_{uu} = d-2$. This can be easily obtained from the Christoffel symbols (C.3).

In the Feynman gauge $\xi = 1$, corresponding to a "minimal" wave operator in the sense of Barvinsky and Vilkovisky [14], one can work out a Schwinger-DeWitt construction[4] and this time we have to deal with bi-vectors. The recurrences are slightly changed to

$$(n+1)\,a_{n+1\,\mu\nu'} + \partial^\rho\sigma\;\nabla_\rho a_{n+1\,\mu\nu'} = \triangle^{-\frac{1}{2}}\Box\,(\triangle^{\frac{1}{2}}a_{n\,\mu\nu'}) - R_\mu^{\ \rho}a_{n\,\rho\nu'}\,, \tag{3.19}$$

and the chain of HaMiDeW coefficients starts with the vector geodesic parallel transporter $a_{0\,\mu\nu'}(x,x') = P_{\mu\nu'}(x,x')$ (appendix C) .

What one can show in this case is that the recurrences are solved by

$$a_{n\,\mu\nu'}(x,x') = \begin{cases} P_{\mu\nu'}(x,x')\,, & n=0\,,\\ (2-d)\delta_{u\mu}\delta_{u'\nu'}\dfrac{\tan\frac{u-u'}{2}}{\frac{u-u'}{2}}\,, & n=1\,,\\ 0\,, & n\geq 2\,. \end{cases} \tag{3.20}$$

[4] As shown by R. Endo [15], having the vector kernel in Feynman gauge one can easily go to any other covariant gauge.

The vector kernel is then

$$K^{\mu}_{\nu'}(x,x'\mid s)=\frac{i\triangle^{\frac{1}{2}}}{(4\pi is)^{\frac{d}{2}}}\;e^{i\sigma/2s}\left(\delta^{\mu}_{\rho}-is\frac{\tan\frac{u-u'}{2}}{\frac{u-u'}{2}}R^{\mu}_{\rho}\right)P^{\rho}_{\nu'}(x,x')\,, \tag{3.21}$$

and the Green function can be written in terms of the massless scalar Green function as

$$G^{\mu}_{\nu'}(x,x')=\left(D(x,x')\,\delta^{\mu}_{\rho}-\frac{1}{4\pi\cos^2\frac{u-u'}{2}}Q(x,x')R^{\mu}_{\rho}\right)P^{\rho}_{\nu'}(x,x')\,, \tag{3.22}$$

where the functional dependence of Q on $u-u'$ and σ is precisely the same as in D but in two dimensions less (see 3.12), i.e.

$$Q(x,x')=\frac{-i\,\Gamma(d/2-2)}{2(2\pi)^{d/2-1}}\left[\frac{u-u'}{\sin(u-u')}\right]^{\frac{d-4}{2}}\left(\frac{1}{\sigma}\right)^{\frac{d-4}{2}}=\frac{-i\,\Gamma(d/2-2)}{2(2\pi)^{d/2-1}}\left(\frac{1}{\Phi}\right)^{\frac{d-4}{2}} \tag{3.23}$$

That we should not expect leading-WKB exactness this time can be seen by examining the coincidence limit $x\to x'$, where general results [11, 12] are available. In particular for the plane wave under consideration, one must have $a_{1\,\mu\nu}(x,x)=-R_{\mu\nu}(x)$, and this can be readily checked in (3.21) remembering that the coincidence limit of the vector parallel transporter is just the metric tensor, $P_{\mu\nu}(x,x)=g_{\mu\nu}(x)$.

After all, we obtained the minimal departure: next-to-leading WKB- exactness. This time, the similarity with flat space results is still present although obscured by an additional term. The flat space limit can be taken as in the preceding two cases, this time the vector parallel transporter goes to the metric tensor and the a_1 coefficient together with the Ricci tensor go to zero to end up with the usual Minkowski space results in Feynman gauge, that is, the metric tensor times the massless scalar propagator.

4. Propagators in *ESU*: resummation and Penrose limit

One can take advantage of the fact that the ESU has the same Penrose limit and try to take the limit directly in the Green functions for ESU where some results are available in the literature. How does the limit work directly on the propagators is apparently not easy to see in the mode summation form. But, after a resummation things might get clearer. The resummation we will explore is the one implicit in the so called "duality spectrum-geodesics". That is, the kernel can be written either as an eigenfunction expansion or as a "sum over classical paths" [16].

4.1 Scalar field in *ESU*

The resummation is implicit in the following form for the scalar Green function in ESU_4 obtained by Dowker and Critchley [17](see also [16]) based on the Schwinger-DeWitt technique. The Schwinger-DeWitt kernel, as well as the heat kernel, factorizes for a product space and since ESU_4 is nothing but $R\times S^3$ one just needs the free kernel for the time direction $K_R(t,t'\mid s)=\frac{i}{(4\pi is)^{1/2}}\;e^{-ia^2(t-t')^2/4s}$ and the kernel for the 3-sphere. The whole problem reduces to finding K_{S^3} and one can show that the 3-sphere is leading-WKB exact.[5] The only complication is that, due to the compactness of the sphere, one has multiple geodesics in addition to the direct one so that one has to include indirect geodesic contributions which restore the periodicity on the sphere

$$K_{S^3}(q,q' \mid s) = \sum_{n=-\infty}^{\infty} K^0_{S^3}(\chi + 2\pi n a \mid s)\,, \tag{4.1}$$

where χ is the length of the shortest arc connecting the two points q, q' on the 3-sphere and

$$K^0_{S^3}(\chi \mid s) = \frac{1}{(4\pi i s)^{\frac{3}{2}}} \triangle^{\frac{1}{2}}\, e^{i\chi^2/4s + is/a^2}\,, \tag{4.2}$$

with the Van Vleck-Morette determinant for the sphere resulting in $\triangle^{\frac{1}{2}} = \frac{\chi/a}{\sin(\chi/a)}$. The corresponding Green function for ESU_4 is also given by direct plus indirect geodesic contributions

$$G_{ESU_4}(x,x') = \sum_{n=-\infty}^{\infty} G^0_{ESU_4}(t-t', \chi + 2\pi n a) \tag{4.3}$$

$$G^0_{ESU_4}(t-t',\chi) = \frac{i\triangle^{\frac{1}{2}}}{8\pi}\left(\frac{m^2 - a^{-2}}{2\sigma}\right)^{\frac{1}{2}} H^{(2)}_1\left(\left[-2(m^2 - a^{-2})\sigma\right]^{\frac{1}{2}}\right), \tag{4.4}$$

where the direct geodetic interval is $\sigma = \frac{-a^2(t-t')^2 + \chi^2}{2}$. Now one can take the Penrose limit (see appendix A), and the result is that only the direct geodesic contribution survives the limit to give precisely the plane wave results (3.10) for $d = 4$. The indirect geodesic terms become rapidly oscillating or exponentially decaying. Therefore they vanish as a distribution for $a \to \infty$. This is similar to the flat space limit of ESU_4 discussed in [17].

This construction can be generalized to higher dimensional ESU_n. For odd-dimensional spheres the Schwinger-DeWitt kernel is WKB exact [16] and for even-dimensional spheres one only has an asymptotic expansion, but in all cases the only term that survives the Penrose limit is the first coefficient in the direct geodesic contribution and this can be seen in the asymptotic expansion, all other terms are suppressed by inverse powers of the radius or are rapidly oscillating.

4.2 Spinor field in ESU

For ESU_4, Altaie and Dowker [18] obtained the spinor S-D kernel and the spinor Green function. To our purposes it suffices to take a look at the spinor S-D kernel, which due to the compactness of the 3-sphere is again a sum over all geodesics connecting the two points, with the direct term

$$\mathbb{K}^0_{ESU_4}(x,x' \mid s) = \frac{i}{(4\pi i s)^2} \triangle^{\frac{1}{2}}\, e^{i(\chi^2 - a^2(t-t')^2)/4s}\left(1 - \frac{is\tan(\chi/a)}{a\chi}\right)\mathbb{U}(x,x')\,. \tag{4.5}$$

[5] The odd-dimensional spheres turn out to be WKB exact after factorizing a constant phase involving the scalar curvature. This phase can in turn be absorbed in the definition of the differential operator and its effect in the Green function is just a shift in the mass. This must be taken into account when comparing the results in [17] with those in [16]

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In the Penrose limit (see appendix A) one gets again the same behavior, i.e. only the first coefficient in the direct geodesic term survive and everything else is suppressed as in the scalar case.

One can follow this construction using the spinor kernel for the higher-dimensional spheres, already calculated by Camporesi [19], and one gets again agreement with our previous results from direct computation in the plane wave background. In all cases, the relevant information is contained in the S-D asymptotic (S-D stands either for Schwinger-DeWitt or for short-distance), the rest is just scaled away in the Penrose limit. This is precisely the resummation we were looking for.

5. Plane wave propagators via Penrose limit of $AdS \times S$

The key tool for the previous results was the resummation implicit in the Schwinger-DeWitt asymptotics. So, this could be the recipe to obtain the limiting values of the Green functions.

Let us first explore for some cases where closed results are still available before drawing conclusions for the generic case.

5.1 $AdS_3 \times S^3$ with equal radii

The kernel for AdS_3 can be obtained from the heat kernel for H^3 [16] by analytic continuation, for spacelike intervals both must coincide. For timelike intervals in AdS_3, which are the relevant ones for the Penrose limit since the null geodesic is always spacelike on the sphere so that it must be timelike in AdS_3 [4], one has the continuation

$$K_{AdS_3}(\zeta \mid s) = \frac{i}{(4\pi i s)^{\frac{3}{2}}} \triangle^{\frac{1}{2}} \, e^{i\zeta^2/4s - is/a^2} \,, \tag{5.1}$$

where $\zeta^2/2$ is the geodetic interval and $\triangle^{\frac{1}{2}} = \frac{\zeta/a}{\sinh(\zeta/a)}$. This kernel gives the standard Green function corresponding to Dirichlet boundary conditions, which can be expressed in terms of hypergeometric functions (see, e.g. [20]).

This allows us to write the exact kernel for $AdS_3 \times S^3$, given again by a sum to produce the periodicity on the 3-sphere, with the direct geodesic term

$$K^0_{AdS_3\times S^3}(\zeta,\chi \mid s) = \frac{i}{(4\pi i s)^3} \frac{\zeta/a}{\sinh(\zeta/a)} \frac{\chi/a}{\sin(\chi/a)} \, e^{i(\zeta^2+\chi^2)/4s} \,. \tag{5.2}$$

In the Penrose limit, the indirect geodesic contributions are suppressed, $\zeta^2+\chi^2 \to 2\sigma$, $\frac{\zeta/a}{\sinh(\zeta/a)}$ and $\frac{\chi/a}{\sin(\chi/a)}$ both $\to \frac{u-u'}{\sin(u-u')}$ and one recovers the plane wave results (3.10) for $d=6$.

5.2 Conformal coupling

In $AdS_{p+1} \times S^{q+1}$ with equal radii which is then conformally flat, for the conformally coupled scalar one gets a powerlike function in the total chordal distance when mapping to the massless scalar in flat space. This can also be obtained by a direct summation of the harmonics on the sphere as shown in [6].[6] The limit agrees with the plane wave result for the massless case where the Green function is an inverse power of Φ (see appendix A).

We can accommodate this case in our scheme. Start with $AdS^3 \times S^3$ and use the whole kernel, that is

$$K_{AdS_3 \times S^3}(\zeta, \chi \mid s) = \sum_{n=-\infty}^{\infty} K^0_{AdS_3 \times S^3}(\zeta, \chi + 2\pi n a \mid s)\,. \tag{5.3}$$

For the Weyl invariant scalar, corresponding in this case to $m = 0$ one can take the proper time integral and perform the sum of all direct and indirect geodesics to get

$$G_{AdS_3 \times S^3}(\zeta, \chi \mid s) \propto \frac{1}{[\cos(\chi/a) - \cos(\zeta/a)]^2} \propto \frac{1}{[\text{total chordal distance squared}]^2}\,. \tag{5.4}$$

Now one can take the Penrose limit at any of the two stages, in this final expression or first in the kernel.

The Weyl coupling case for higher dimension can now be generated by the "intertwining" technique [16]. Applied to the kernel one obtains a kernel that produces the desired power in the total chordal distance for the Green function. Alternative, the intertwining can be applied directly to the Green function. The intertwining technique reduces basically to the fact that something missing one can obtain the kernel or the Green function for the conformally coupled scalar by just taking derivatives with respect to the chordal distances. One can start with $AdS_3 \times S^1$, taking partial derivative with respect the chordal distance in AdS one gets the results for the product space with two dimensions higher in AdS and taking partial derivative with respect the chordal distance in the sphere one gets the results for the product space with two dimensions higher in the sphere.[7] This way one generates the higher negative powers in the total distance for the conformally coupled scalar [6]. Again, in the Penrose limit only the leading term of the direct geodesic survives the limit.

6. Conclusions

Our main result is the explicit construction of the spinor and vector propagator in the plane wave background (2.1) arising in a Penrose limit from $AdS_{p+1} \times S^{q+1}$. The spinor propagator is constructed for generic mass values, the vector propagator for massless gauge fields in Feynman gauge.

The construction was based on the Schwinger-DeWitt technique. In general backgrounds via this method one gets only an asymptotic WKB series with respect to the approach to the light cone. Global issues for the propagators remain open. For the background under discussion we could show that the series terminates with its leading or next-to-leading term. This then strongly suggests that the resulting expressions are indeed the

[6] In fact, in [6] one also obtains a powerlike function for a particular mass in the case where the radii are different, when no conformal map to flat space is possible. We have managed also to reproduce this result using the kernels and a nice relation to the conformal situation was found in terms of a contour integral, see appendix D.

[7] To cover he whole range of dimensions for the product space $AdS \times S$ ([odd,odd], [odd,even], [even,odd] and [even,even]) one needs in addition the S^2 and AdS_2 results, see [16].

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correct propagators. We checked this by reproducing the scalar propagator already constructed in the literature by different methods. In this check we also explained by the WKB exactness the structural similarity with the flat space scalar propagator pointed out in [5]. The propagator in both cases is given by the same function of the respective geodesic distances up to an additional factor generated by the nontrivial Van Vleck-Morette determinant of the plane wave background. This ordinary determinant for the plane wave can be shown to be equal to the functional determinant of the quadratic fluctuations in the path integral formalism [21], where leading-WKB-exactness amounts to the exactness of the gaussian approximation for the path integral.

Besides the explicit construction in the plane wave geometry, we made some observations on the relation between both propagators and kernels to those in spaces from which the plane wave arises in a Penrose limit. After remarking that the plane wave under study can also be obtained from ESU, we discussed the limit starting from known explicit expressions both for the scalars and spinors in ESU. It turned out that only the leading term in the direct geodesic contribution survives the limit. This nicely corresponds with the local nature of the Penrose limit. This picture was supported by similar observations starting from some special $AdS \times S$ cases. In addition for the $AdS \times S$ propagators we were able to explain the distinguished role of a special mass value for non-Weyl invariant coupling in spaces with different radii for AdS_{p+1} and S^{q+1} [6]. Just for this value in the exponent of the Schwinger-DeWitt kernel the term linear in the proper time cancels and one can explicitly perform the sum. A contour integral relates the kernels and propagators for this special non-conformally flat (conformal to a spacetime with a conical singularity) case to a conformal flat, as shown in appendix D.

Further study should clarify whether there is a general theorem behind. Given a generic plane wave arising in a Penrose limit from some other spacetime, does then the information on the first few coefficients of the direct geodesic contribution in the original spacetime always contain enough information to get the plane wave propagators? Is the WKB-exactness a generic feature of the Penrose limit?

Of special interest would also be to find a relation between the WKB-exactness of the field theoretic propagators on the plane wave (2.1) and the successful semiclassical description of strings in this background [22, 23].

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A. Geodesic and chordal distances in ESU_{n+1}

Embedding the n-sphere in $(n+1)$-euclidean space, a point on the sphere is given by the vector $a\,\hat{\Omega}$, with

$$\hat{\Omega} = (\cos\alpha\ \cos\beta\,,\ \cos\alpha\ \sin\beta\,,\ \sin\alpha\ \hat{\omega}) \tag{A.1}$$

where $\hat{\omega}$ is a unit vector on the $(n-2)$-sphere and the parametrization is as in equation (2.4). The chordal distance squared $\mu_n(x,x')$ between two points x and x' is related to the arc $\chi_n(x,x')$ (direct geodesic distance) by

$$1-\frac{\mu_n}{2a^2}=\cos\frac{\chi_n}{a}=\cos\alpha\,\cos\alpha'\,\cos(\beta-\beta')+\sin\alpha\,\sin\alpha'\,\cos\frac{\chi_{n-2}}{a} \tag{A.2}$$

where $\cos\frac{\chi_n}{a}\equiv\hat{\Omega}\cdot\hat{\Omega}'$ and $\cos\frac{\chi_{n-2}}{a}\equiv\hat{\omega}\cdot\hat{\omega}'$, being χ_{n-2} the arc along the $(n-2)$-sphere. Let us take for simplicity x' to be at the origin.

Going to the local coordinates (equation 2.6) and expanding in inverse powers of the radius

$$\cos\frac{\chi_n}{a}=\cos\alpha\,\cos\beta=\cos u-\frac{\Phi}{a^2}+O(a^{-4}) \tag{A.3}$$

$$\frac{\chi_n}{a}=u+\frac{\Psi}{a^2}+O(a^{-4})\,, \tag{A.4}$$

where

$$\Phi=v\sin u+\frac{\vec{x}^2}{2}\cos u \tag{A.5}$$

$$\Psi=v+\frac{\vec{x}^2}{2}\cot u\,. \tag{A.6}$$

These two quantities naturally arise in the plane wave, $u\Psi$ is the geodetic interval [5, 2] and 2Φ is the limiting value of the total chordal distance squared in $AdS\times S$ as elucidated in [6]. Going back to our ESU, it is easy to see that this also holds provided one compactifies the time into a circle so that t becomes an angle. That is, as $a\to\infty$ one has

$$\text{geodetic interval}=\frac{-a^2t^2+\chi^2}{2}\to u\Psi \tag{A.7}$$

$$\frac{\text{total chordal distance squared}}{2}=-a^2[1-\cos t]+\frac{\mu}{2}\to\Phi\,. \tag{A.8}$$

B. Spinor geodesic parallel transporter

Let us go to the frame given by

$$ds^2=2\theta^+\theta^-+\vec{\theta}\cdot\vec{\theta}\equiv\eta_{ab}\theta^a\theta^b \tag{B.1}$$

$$\theta^+=du\,,\qquad\theta^-=dv-\frac{1}{2}\vec{x}^2du\,,\qquad\vec{\theta}=d\vec{x}\,. \tag{B.2}$$

The spin-connection components can be read off from the first Cartan structure equation

$$d\theta^a+\omega^a_{\ b}\wedge\theta^b \tag{B.3}$$

(tangent indices $a,b=+,-,i$ with $i=1,\ldots,d-2$ being the transverse ones) and the only nonvanishing ones are

$$\omega^{i-}=-\omega^{-i}=x^i\,du\,. \tag{B.4}$$

The covariant derivative on spinors

$$\nabla_\mu \equiv \partial_\mu + \Gamma_\mu = \partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_a\gamma_b\,, \tag{B.5}$$

where the $\gamma' s$ fulfill the Clifford algebra in tangent space

$$\{\gamma_a,\, \gamma_b\} = 2\eta_{ab}\,\mathbb{I}\,, \tag{B.6}$$

is found to be

$$\nabla_\mu = \begin{cases} \partial_u - \frac{1}{2}\gamma_- \vec{\gamma}\cdot\vec{x} \\ \partial_v \\ \partial_i \end{cases} \tag{B.7}$$

that is, only Γ_u is nonzero. An important property is that $(\Gamma_u)^2 = 0$ because $(\gamma_-)^2 = \mathbb{I}\,\eta_{--} = 0$, i.e. Γ_u is nilpotent.

The spinor D'Alembertian can be written in terms of the scalar one as

$$g^{\mu\nu}\nabla_\mu\nabla_\nu = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu) + 2\Gamma_u\partial_v\,. \tag{B.8}$$

The spinor parallel transporter is a bi-spinor that parallel transports a spinor along a given path and the path we need is the geodesic connecting the two points. This spinor geodesic parallel transporter must satisfy the parallel transport equation and the initial condition

$$\partial^\mu\sigma\;\nabla_\mu\mathbb{U}(x,x') = 0\,, \qquad \mathbb{U}(x,x) = \mathbb{I}\,. \tag{B.9}$$

One can write a Dyson-type representation for it (see, e.g. [19]), integrating along the geodesic emanating from x' [4]

$$\mathbb{U}(t) = \mathbf{P}\exp - \int_0^t \Gamma_\mu(\tau)dx^\mu(\tau)\,. \tag{B.10}$$

But for the plane wave metric, due to the nilpotency of Γ_μ, one can drop the path ordering symbol $\mathbf{P}$ because the matrices in the exponent commute, therefore one can perform the integration to get

$$\mathbb{U}(x,x') = \exp\,\frac{1}{2}\gamma_-\vec{\gamma}\cdot(\vec{x}+\vec{x}')\tan\frac{u-u'}{2} = \mathbb{I} + \frac{1}{2}\gamma_-\vec{\gamma}\cdot(\vec{x}+\vec{x}')\tan\frac{u-u'}{2}\,. \tag{B.11}$$

Finally, one can easily check that $\Box\,\mathbb{U}(x,x') = 0$.

C. Vector geodesic parallel transporter

The Christoffel symbols for the plane wave metric can be directly read off from the geodesic equations which in turn can be derived from the lagrangian

$$L(\dot{u},\dot{v},\dot{\vec{x}},\vec{x}) = \frac{1}{2}\dot{x}^\mu\dot{x}_\mu = \dot{u}\dot{v} + \frac{1}{2}\dot{\vec{x}}^2 - \frac{1}{2}\dot{u}^2\,\vec{x}^2\,, \tag{C.1}$$

where the dots are derivatives with respect to an affine parameter along the geodesic. The geodesic equations read

$$\ddot{u} = 0 \tag{C.2a}$$
$$\ddot{v} - 2\vec{x}\cdot\dot{\vec{x}}\dot{u} = 0 \tag{C.2b}$$
$$\ddot{\vec{x}} + \dot{u}^2\vec{x} = 0 \tag{C.2c}$$

and therefore the only nonzero Christoffels are

$$(\Gamma_u)^i{}_u = x^i\,, \qquad (\Gamma_u)^v{}_i = (\Gamma_i)^v{}_u = -x^i\,. \tag{C.3}$$

There are two types of geodesics [4]: type-A when $\dot{u} = 0$ and the null ones in this category are parallel to the propagation direction of the wave, and type-B when one can take u as the affine parameter which is the generic situation. For this generic case, the lagrangian (C.1) is $\frac{1}{2}\dot{x}^\mu\dot{x}_\mu = const$ and reproduces the one for a harmonic oscillator of unit mass and unit frequency plus an extra $\dot{v}$ term. Then, it is not difficult to see that the recipe to get the geodetic interval between two generic points is just the replacing by the classical action for the oscillator between two points $\vec{x}$ and $\vec{x}'$ followed by the shifts $u \to u - u'$ and $v \to v - v'$, so that

$$\text{geodetic interval} = (u-u')(v-v') + (u-u')\left[\frac{\vec{x}^2 + \vec{x}'^2}{2}\cot(u-u') - \vec{x}\cdot\vec{x}'\csc(u-u')\right] \tag{C.4}$$

and for type-A, one just has to let $u \to 0$ which simply produces

$$\text{geodetic interval} = \frac{(\vec{x}-\vec{x}')^2}{2}\,. \tag{C.5}$$

This recipe also works for the quantities Ψ, Φ and $\triangle$, previously defined.

The vector parallel transporter is a bi-vector that parallel transports a vector along a given path, and the path we need is the geodesic connecting the two points. This vector geodesic parallel transporter must satisfy the parallel transport equation and the initial condition

$$\partial^\rho\sigma\,\nabla_\rho P_{\mu\nu'}(x,x') = 0\,, \qquad P_{\mu\nu}(x,x) = g_{\mu\nu}(x)\,. \tag{C.6}$$

One can also write a Dyson-type representation for it (see, e.g. [24]), integrating along the geodesic emanating from x' [4]

$$P^\mu_{\nu'}(x,x') = \mathbf{P}\exp -\int_0^t (\Gamma_\rho)^\mu{}_{\nu'}(\tau)\,dx^\rho(\tau)\,. \tag{C.7}$$

But for the plane wave metric one can check that Γ_ρ as a matrix, with μ and ν' labeling its rows and columns respectively, commutes with itself at different points. One can therefore drop the path ordering symbol and perform the integration to get

$$P^\mu_{\nu'}(x,x') = \exp\begin{pmatrix} 0 & 0 & \vec{0} \\ \frac{\vec{x}^2-\vec{x}'^2}{2} & 0 & (\vec{x}+\vec{x}')\tan\frac{u-u'}{2} \\ -(\vec{x}^{\,t}+\vec{x}'^{\,t})\tan\frac{u-u'}{2} & \vec{0}^{\,t} & \mathbb{O} \end{pmatrix} \tag{C.8a}$$

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$$= \begin{pmatrix} 1 & 0 & \vec{0} \\ \frac{\vec{x}^2 - \vec{x}'^2}{2} - \frac{|\vec{x}+\vec{x}'|^2}{2} \tan^2 \frac{u-u'}{2} & 1 & (\vec{x}+\vec{x}') \tan \frac{u-u'}{2} \\ -(\vec{x}+\vec{x}') \tan \frac{u-u'}{2} & \vec{0}^t & \mathbb{I} \end{pmatrix} . \tag{C.8b}$$

D. Non-conformally flat background

Let us consider the euclidean version $H^3 \times S^3$ with different radii (say, a and αa). Up to normalization factors, the kernel $K^*_{H^3 \times S^3}(\zeta, \chi \mid s)$ is given by

$$\frac{1}{s^3} \frac{\zeta/a}{\sinh(\zeta/a)} \frac{1}{\sin(\chi/\alpha a)} e^{is/a^2(1/\alpha^2 - 1)} \sum_{n=-\infty}^{\infty} (\chi/\alpha a \, + \, 2\pi n) \, e^{i[\zeta^2 + (\chi + 2\pi n \alpha a)^2]/4s} . \tag{D.1}$$

The kernel this time has a remaining "s" dependent term in the exponent that can only be eliminated by a special value of the mass, $m_*^2 = \frac{1}{a^2}(1 - \frac{1}{\alpha^2})$. This value of the mass is precisely the one used in [6] to get a closed expression for the Green function. What one can see is that for this value one can perform the integral to get for the Green function

$$\frac{\zeta/a}{\sinh(\zeta/a)} \frac{1}{\sin(\chi/\alpha a)} \sum_{n=-\infty}^{\infty} \frac{\chi/\alpha a \, + \, 2\pi n}{[\zeta^2 + (\chi + 2\pi n \alpha a)^2]^2} \tag{D.2}$$

and the resulting series can be exactly computed with the aid of a Poisson summation (after taking partial derivative with respect to x to relate both sums)

$$\sum_{n=-\infty}^{\infty} \frac{y}{y^2 + (x+n)^2} = \frac{1 - e^{-4\pi y}}{1 - 2\cos(2\pi x) e^{-2\pi y} + e^{-4\pi y}} \tag{D.3}$$

to get

$$G^*_{H^3 \times S^3}(\zeta, \chi) \propto \frac{\sinh(\zeta/\alpha a)}{\sinh(\zeta/a)} \frac{1}{[\cosh(\zeta/\alpha a) - \cos(\chi/\alpha a)]^2} . \tag{D.4}$$

Now, when the two radii are equal ($\alpha = 1$) one gets of course the conformally coupled scalar in the conformally flat background. The conformally flat case is periodic on the arc in the sphere χ with period $2\pi a$ while the period in the non-conformally flat is $2\pi\alpha a$. The interesting thing to notice is that the kernels as well as the Green functions are related by a contour integral due to Sommerfeld that restores the appropriate periodicity (see, e.g. [25]). This can be explicitly checked for the special mass above [26]

$$G^*_{H^3 \times S^3}(\zeta, \chi) = G_{H^3 \times S^3}(\zeta, \chi) + \frac{i}{4\pi\alpha} \int_\Gamma dw \, \cot\left(\frac{w}{2\alpha}\right) G_{H^3 \times S^3}(\zeta, \chi + wa) , \tag{D.5}$$

where the contour Γ consists of two vertical lines from $(-\pi + i\infty)$ to $(-\pi - i\infty)$ and from $(\pi - i\infty)$ to $(\pi + i\infty)$ and intersecting the real axis between the poles of $\cot(\frac{w}{2\alpha})$: $-2\pi\alpha, 0$ and $0, 2\pi\alpha$, respectively.

This very same formula gives the heat kernel for the cone starting with the one for the plane. This is a remarkable property since equal radii corresponds to a conformally flat situation and different radii is conformal to a singular background with a tip, a conical singularity.

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